Nonlinear surface waves at the interfaces of left-handed electromagnetic media

S. A. Darmanyan,[1,*] M. Nevière,[2] and A. A. Zakhidov[3]
[1]Institute of Spectroscopy, Russian Academy of Sciences, 142190 Troitsk, Russia
[2]Institut Fresnel, Faculté des Sciences et Techniques de Saint Jérôme, 13397 Marseille, Cedex 20, France
[3]UTD-Nanotech Institute, University of Texas at Dallas, Dallas, Texas 75083-0688, USA
(Received 14 December 2004; revised manuscript received 21 July 2005; published 21 September 2005)

We report the existence of families of nonlinear TE-polarized surface waves propagating along the interface between different conventional and left-handed electromagnetic media as well as between two left-handed media. Both nonlinear/nonlinear and linear/nonlinear interfaces are considered. The constraints for the mode existence are identified and the energy flow associated with the surface modes is calculated.

DOI: 10.1103/PhysRevE.72.036615 PACS number(s): 42.65.Tg, 78.68.+m

I. INTRODUCTION

Recently, left-handed electromagnetic materials (LHMs) have been attracting a great deal of interest due to both the first encouraging experimental observations and tremendous prospects for applications [1–10]. LHMs, i.e., materials with simultaneously negative dielectric permittivity $\varepsilon(\omega)<0$ and magnetic permeability $\mu(\omega)<0$ in a certain frequency range exhibit *negative refractive index* and obey the anomalous Snell's law. In LHMs many exciting and unusual properties are expected. For example, the Doppler effect should be opposite to that of usual matter, the Cherenkov radiation cone should direct in the backward rather than forward direction, etc. [1].

The existence of LHMs was predicted by Veselago over 35 years ago [1]. The concept of an LHM has first been experimentally proven for microwaves by Smith and Kroll [2], who created a composite LHM made of arrays of split ring resonators embedded into a metallic mesh. In the following years, the existence and properties of LHMs in the microwave domain were experimentally studied by different groups, see, e.g., [3–6]. Recently, it was demonstrated that the rectangular slab of an LHM can act as a new type of lens where resolution is not restricted by the diffraction limit [6]. A review of recent achievements in the field can be found, e.g., in [7].

It has also been understood that the general properties of LHM such as negative refraction (NR) can be achieved not only in a classical Veselago-type LHM with simultaneously negative $\varepsilon(\omega)$ and $\mu(\omega)$ but also in other classes of materials in which the group and phase velocities have opposite signs [8,9]. Such materials are generically called negative-index materials (NIMs). One class of NIMs is photonic crystals [10] in which NR has been predicted and already observed in both microwave and recently in the ir frequency bands [11]. Another class of NIMs includes materials with bands of exciton-polaritons [12] or plasmon-polaritons [13] with negative dispersion, or in other words, with negative effective masses in certain frequency ranges [12,14]. In addition, even in highly lossy materials the NR can still be observed. In fact, the range of frequencies for its existence, or the NR bandwidth, can be increased on account of losses [15].

In NIMs nonlinearity can lead to many interesting effects. For instance, due to the opposite direction of wave vectors for the fundamental frequency and higher harmonics the generation of harmonics is most intense in the reflected geometry. This unusual behavior is opposite to that in conventional nonlinear-optical materials. Moreover, the lensing effect for higher harmonics may take place, even though for the fundamental frequency no lensing exists [12] and the nonlinear superlens effect can be achieved with even higher resolution than for the linear superlens. Possible nonlinear effects in a bulk LHM were discussed in [14,16].

Surface waves (SWs) propagate along the interface and decay in the transverse direction. Since SWs have the field maximum at or near the interface, they are a very sensitive and convenient tool for studying physical properties of surfaces. The theory and application of SWs to the diagnostics of surfaces and thin films is described, e.g., in [17]. In Refs. [18–21] the surface polaritons propagating along the interface between a linear LHM and different types of conventional right-handed materials (RHMs) were studied. The domains of existence and some peculiar features of TM- and TE- polarized waves were identified. It is worth noting that SWs are particularly important for the lensing effect mentioned above since the amplification of evanescent modes is responsible for the subwavelength resolution. For instance, in recent papers [22,23] it was shown that SWs have a strong impact on the image resolution of an LHM flat lens. In particular, the presence of SWs leads to an additional limitation of the image resolution and degrades the image quality. Therefore the study of possible effects at the interfaces of materials including materials with strong optical nonlinearities is of significant importance. The existence of nonlinear TE modes at the interface between LHM and RHM has been shown in [24].

To further study the interesting properties of SWs in LHMs, in this paper we will consider nonlinear surface electromagnetic waves at the interface between LHMs and various types of conventional solids. We will show that along with SWs studied in [24] there are families of SWs that exist in complementary domains of system parameters. The re-

*Present address: Science and Technology Division, Corning Incorporated, SP-TD-01, Corning, NY 14831. Email address: darmanyasa@corning.com

mainder of the paper is organized as follows. In the following section we introduce the model of the system under consideration. In the subsequent sections, III–V, we find new families of SWs and study their features for different interfaces between nonlinear media, namely, defocusing-defocusing, defocusing-focusing, and defocusing-linear. The cases of focusing-focusing and focusing-linear media interfaces have been studied in [24]. We also note that, in contrast to [24], by only utilizing singular solutions of governing nonlinear equations we prove the existence of SWs in the case of defocusing media.

II. THE MODEL

In what follows, we study the existence of the TE-polarized nonlinear surface wave propagating along the interface between both left- and right-handed media and between two left-handed media. We consider waves propagating along the x axis located at the interface ($z=0$) between two semi-infinite media. Each medium is characterized by linear electric permittivity ε_i, magnetic permeability μ_i and parameter α_i, that describes Kerr-type nonlinearity [$\boldsymbol{D}^{\mathrm{NL}}=\alpha_i \boldsymbol{E}(\boldsymbol{E}\boldsymbol{E}^*)$], where $i=1$ for $z>0$ and $i=2$ for $z<0$. For simplicity, we assume that the contacting media are lossless and isotropic.

To find the surface TE-polarized waves, we substitute the electric and magnetic fields in the form $E=(0,E_y,0)e^{jqx-j\omega t}$ +c.c. and $H=(H_x,0,H_z)e^{jqx-j\omega t}$+c.c. into Maxwell equations and arrive at the following governing equation for the y-component of the electric field amplitude [$A(z)=E_y(z)$]:

$$\frac{\partial^2 A_i}{\partial z^2} - \eta_i^2 A_i + \chi_i |A_i|^2 A_i = 0, \qquad (1)$$

where $\eta_i^2=q^2-\varepsilon_i(\omega)\mu_i(\omega)$, $\chi_i=\mu_i\alpha_i$. In Eq. (1) the dimensionless variables $z\rightarrow k_0 z, q\rightarrow q/k_0$ are introduced, $k_0=\omega/c$.

Equation (1) possesses the following solitonlike solutions

$$A_i = \frac{a_i}{\cosh \eta_i(z-z_i)}, \quad a_i = \pm \eta_i\sqrt{2/\chi_i}, \qquad (2a)$$

$$A_i = \frac{a_i}{\sinh \eta_i(z-z_i)}, \quad a_i = \pm \eta_i\sqrt{-2/\chi_i}, \qquad (2b)$$

that we will utilize in what follows to construct the nonlinear surface waves. Equations (2a) and (2b) are valid for $\chi_i>0$ and $\chi_i<0$, respectively, and for $\eta_i^2\geqslant 0$. This means that, e.g., in the case of $\alpha_i>0$ for the left-handed media ($\mu_i<0$) the solution (2b) satisfies Eq. (1), while for right-handed media ($\mu_i>0$) the solution (2a) applies (as in the standard case of self-focusing nonlinearity). Without loss of generality, we choose $\eta_i>0$.

We note that the solution (2b) at $z=z_i$ diverges. Nevertheless, it can be used for constructing surface modes provided that the singularity point is located outside the corresponding medium. Such an approach was used in [25,26] to find SW solutions in RHM with cubic and quadratic nonlinearities.

To obtain the surface mode solution one needs to satisfy the boundary conditions (BCs) for the electromagnetic field at the interface (plane $z=0$) which for the TE waves read as

$$A_1 = A_2,$$

$$\frac{1}{\mu_1}\frac{dA_1}{dz} = \frac{1}{\mu_2}\frac{dA_2}{dz}. \qquad (3)$$

Solutions (2) have three free parameters z_1, z_2, and q. Two of these parameters are determined by the BC equations (3). Thus the TE nonlinear surface waves are one-parametric solutions with the frequency defined by the field source. Let us consider different possible combinations of contacting media.

III. INTERFACE BETWEEN TWO SELF-DEFOCUSING MEDIA ($\chi_1<0, \chi_2<0$)

Substituting Eq. (2b) into BCs (3) we arrive at the following set of equations:

$$\frac{a_1}{\sinh \eta_1 z_1} = \frac{a_2}{\sinh \eta_2 z_2}, \qquad (4a)$$

$$\frac{\eta_1}{\mu_1 a_1}\cosh \eta_1 z_1 = \frac{\eta_2}{\mu_2 a_2}\cosh \eta_2 z_2. \qquad (4b)$$

Equation (4a) was used when writing Eq. (4b). In order to avoid singularities, the constraints $z_1<0$ and $z_2>0$ must be satisfied. Then, according to Eqs. (4) $\mathrm{sgn}(a_1)=-\mathrm{sgn}(a_2)$ and $\mathrm{sgn}(\mu_1)=-\mathrm{sgn}(\mu_2)$. It is convenient to introduce new variables as $\chi=\chi_2/\chi_1$, $\varepsilon=\varepsilon_2/\varepsilon_1$, $\mu=\mu_2/\mu_1$, $\alpha=\alpha_2/\alpha_1$, $\eta=\eta_2/\eta_1$. Then Eqs. (4) can be rewritten in the form

$$\coth \eta_1 z_1 = -\sqrt{\frac{\eta^2-\chi}{\mu^2-\chi}}, \quad \coth \eta_2 z_2 = \frac{\mu}{\eta}\coth \eta_1 z_1 \qquad (5)$$

that defines positions of singularities z_1 and z_2 in terms of the system's parameters. Equations (5) are valid provided that the relations

$$\eta^2 \geqslant \mu^2 > \chi \qquad (6a)$$

$$\text{or } \eta^2 \leqslant \mu^2 < \chi \qquad (6b)$$

hold true. When $\eta^2\rightarrow\mu^2$ we get $z_1\rightarrow-\infty$, $z_2\rightarrow\infty$ and the field amplitude at the interface approaches zero (linear limit). When $\mu^2\rightarrow\chi$ we get $z_1\rightarrow-0, z_2\rightarrow 0$ and the field amplitude at the interface infinitely increases. In the latter case, nonlinear terms additional to the Kerr-type term in the expression for $\boldsymbol{D}^{\mathrm{NL}}$ must be taken into account, which we do not consider here.

A. Energy flow

To better understand physics of wave processes involving SWs in left-handed media, we proceed with calculation of energy flow along the interface. For this, we calculate the Poynting vectors $\mathbf{S}=(c/8\pi)\mathrm{Re}(\mathbf{E}\times\mathbf{H}^*)$ associated with surface modes.

The time-averaged Poynting vector of the surface wave is directed along the x axis ($S_{zi}=0$) and it amounts to

$$S_1 = \frac{cq}{8\pi\mu_1}A_1^2(z), \quad S_2 = \frac{cq}{8\pi\mu_2}A_2^2(z) \tag{7}$$

in medium 1 and medium 2, respectively. As can be inferred from Eq. (7), the guided energy flows have opposite directions in the adjacent media because of the condition $\mathrm{sgn}(\mu_1) \neq \mathrm{sgn}(\mu_2)$. Let us choose $\mu_1 > 0$, $\mu_2 < 0$. Then we get $S_1 > 0$, $S_2 < 0$, i.e., the energy flow in medium 2 is opposite to the phase velocity which in our case is along the positive direction of the x axis. The total energy flow associated with the whole mode is determined by integration over the z coordinate

$$\langle S\rangle = \int_0^\infty S_1 dz + \int_{-\infty}^0 S_2 dz$$
$$= \frac{cq\eta_2}{4\pi\mu_2\chi_2}\left(1 + \frac{\chi\mu}{\eta} + \frac{\mu(\chi-1)}{\eta}\coth \eta_1 z_1\right). \tag{8}$$

Thus the direction of the total energy flow is regulated by the ratio between the two components of the energy flows $\langle S_1\rangle$ and $\langle S_2\rangle$. A similar competition between the two energy flows takes place in the case of the interface between linear media, where in RHMs the total energy flow always has a positive direction while in the LHM case it is negative [21].

B. Domain of existence

We have chosen $\mu_1 > 0$ and $\mu_2 < 0$, therefore the formulas (5)–(8) are valid for interfaces between any media with $\alpha_1 < 0$ and $\alpha_2 > 0$. In spite of the positivity of the nonlinear Kerr coefficient $\alpha_2 > 0$, medium 2 is self-defocusing because of the condition $\chi_2 < 0$ [16]. Thus the SWs obtained above propagate along the interface between two self-defocusing media.

Let us proceed with the case of $\varepsilon_2 < 0$, i.e., consider medium 2 to be left-handed. To simplify the analysis of the inequalities (6), we consider the case of $|\alpha_1| = \alpha_2$. Then $\chi = |\mu|$, and the inequalities (6) can be rewritten as

$$q^2 < q_{cr}^2 = \varepsilon_1\mu_1(\mu^2 - \varepsilon\mu)/(\mu^2 - 1) \tag{9}$$

with $|\mu| > 1$ for Eq. (6a) and $|\mu| < 1$ for Eq. (6b). The condition of compatibility of Eq. (9) with inequalities $q^2 \geqslant \varepsilon_i\mu_i$ gives the following additional constraints: (a) $|\mu||\varepsilon| < 1$ with $\varepsilon_1 > 0$ in case (6a) [$\varepsilon_1 < 0$ is not compatible with Eq. (9) in case (6a)] (b) $|\mu||\varepsilon| > 1$ with $\varepsilon_1 > 0$ or (c) $\varepsilon_1 < 0$ in case (6b). Finally, we arrive at the conclusion that the SWs at the LHM/RHM interface exist provided that either of the three following conditions

$$\varepsilon_1\mu_1 \leqslant q^2 \leqslant q_{cr}^2, \quad |\mu| > 1, \quad \varepsilon\mu < 1, \quad \varepsilon_1 > 0 \tag{10a}$$

$$\varepsilon_2\mu_2 \leqslant q^2 \leqslant q_{cr}^2, \quad |\mu| < 1, \quad \varepsilon\mu > 1, \quad \varepsilon_1 > 0 \tag{10b}$$

$$\varepsilon_2\mu_2 \leqslant q^2 \leqslant q_{cr}^2, \quad |\mu| < 1, \quad \varepsilon_1 < 0 \tag{10c}$$

is satisfied. The transverse profile of the SWs has a maximum at the interface as is shown in Fig. 1(a).

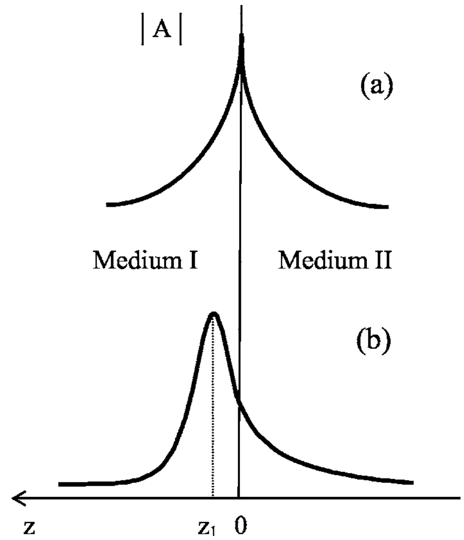

FIG. 1. The sketch of possible SW profiles.

Now it is worthwhile to analyze the expression for the energy flow (8) in more detail. The analysis is convenient to perform in the plane of variables $\{\eta, |\mu|\}$. For this, we rewrite Eq. (8) in the form

$$\langle S\rangle = \frac{cq\eta_1}{4\pi\alpha_2\mu_2^2}f(\eta,\mu), \quad f = (\eta - \mu^2 + \sqrt{(\mu^2 - |\mu|)(\eta^2 - |\mu|)}),$$

where the factor in front of the function $f(\eta, |\mu|)$ is positive. Thus the $\mathrm{sgn}(\langle S\rangle) = \mathrm{sgn}(f)$. An analysis of the function $f(\eta, |\mu|)$ shows that the energy flow $\langle S\rangle$ is positive under conditions (6a), i.e., for $\eta > |\mu| > 1$, while under conditions (6b) $\langle S\rangle$ is positive in the case $\eta_{cr} < \eta < |\mu| < 1$, where $\eta_{cr} = \mu(\mu^2 - \mu - 1)/(\mu^2 + \mu - 1)$ and $\langle S\rangle$ is negative for $\eta < \eta_{cr} < |\mu| < 1$. The latter inequalities can be satisfied only when $(\sqrt{5}-1)/2 \approx 0.62 < |\mu| < 1$. For example, for $|\mu| = 0.8$ we have $\eta_{cr} \approx 0.3$. We remind that parameter $\eta = \eta_2/\eta_1$ describes a relative width of the field distribution on both side of the interface. In Fig. 2 we show the domain of SWs' existence in the plane $\{\eta, |\mu|\}$ (shaded regions), where labels $\langle S\rangle > 0$, $\langle S\rangle < 0$ indicate the direction of the energy flow in corresponding regions. We note that on the line defined as $\eta = \eta_{cr}, |\mu| \leqslant 1$, and $\eta = |\mu| \geqslant 1$ (dashed bold line in Fig. 2) the

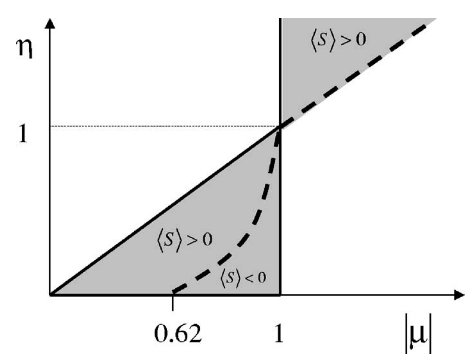

FIG. 2. The domain of the SW existence (shaded regions), case $\chi_1 < 0$, $\chi_2 < 0$.

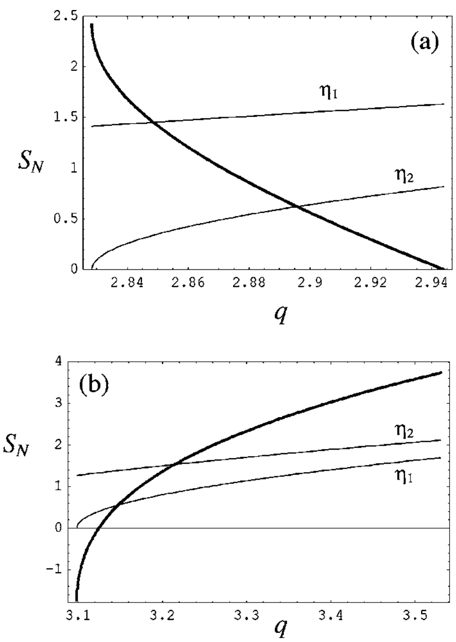

FIG. 3. The normalized energy flow (thick line) and widths $\eta_{1,2}$ (thin lines) as a function of the normalized wave number q, (a) $|\mu|>1$, and (b) $|\mu|<1$. The other relevant parameters are described in the text.

corresponding SW does not carry energy along the interface due to the mutual balance of energy flows in the adjacent media. When $\eta \to |\mu| \geq 1$ the SW amplitude $A(z=0)$ tends to zero ($|z_{1,2}| \to \infty$), however, in the case $\eta = \eta_{cr}, |\mu| \leq 1$ the amplitude has a finite value. Such waves cannot exist in the linear case [21,24]. To further illustrate features of SWs we show in Figs. 3(a) and 3(b) the normalized energy flow $S_N = \langle S \rangle / S_0$, where $S_0 = c/4\pi\alpha_2$ and the width parameters $\eta_{1,2}$ as a function of normalized wave number q for two particular sets of media parameters (a) $\mu_1=1$, $\varepsilon_1=8$, $\mu_2=-2$, $\varepsilon_2=-3$, i.e., $1<|\mu|=2$ and (b) $\mu_1=1$, $\varepsilon_1=8$, $\mu_2=-0.8$, $\varepsilon_2=-12$, i.e., $1>|\mu|=0.8$. In accordance with Eqs. (10a) and (10b) and Fig. 2 the energy flow in case (a) is positive in the domain of SWs existence $\sqrt{8} \leq q \leq q_{cr} = \sqrt{26/3}$ (or $\infty > \eta \geq |\mu| = 2$), while in case (b) in the domain of SWs existence $\sqrt{9.6} \approx 3.1 \leq q \leq q_{cr} \approx 3.53$ (or $0 \leq \eta \leq |\mu| = 0.8$) the energy flow can be of either sign with zero value reached at $q \approx 3.1$ inside the domain.

IV. INTERFACE BETWEEN SELF-FOCUSING AND SELF-DEFOCUSING MEDIA ($\chi_1>0, \chi_2<0$)

In this case we study modes constructed with the combination of the solutions (2a) and (2b). The boundary conditions lead to equations

$$\frac{a_1}{\cosh \eta_1 z_1} = \frac{-a_2}{\sinh \eta_2 z_2}, \tag{11a}$$

$$\frac{\eta_1}{\mu_1} \tanh \eta_1 z_1 = -\frac{\eta_2}{\mu_2} \coth \eta_2 z_2. \tag{11b}$$

As in the previous case, the parameter z_2 should be taken positive ($z_2>0$), while parameter z_1 can be of either sign. It follows from Eqs. (11) that $\mathrm{sgn}(a_1) = -\mathrm{sgn}(a_2)$ and $z_1>0$ at $\mathrm{sgn}(\mu_1) = -\mathrm{sgn}(\mu_2)$ [the mode profile is shown in Fig. 1(b)] or $z_1<0$ at $\mathrm{sgn}(\mu_1) = \mathrm{sgn}(\mu_2)$ [this mode profile is similar to the profile shown in Fig. 1(a)]. Thus at the interface between self-focusing and self-defocusing media the propagation of special new types of SWs with the field maximum located both at the interface and shifted into the focusing medium is possible. As it can be inferred from Eqs. (11) the parameters $z_{1,2}$ are defined by the equations

$$\tanh \eta_1 z_1 = \pm \sqrt{\frac{\eta^2 + |\chi|}{\mu^2 + |\chi|}}, \quad \coth \eta_2 z_2 = -\frac{\mu}{\eta} \tanh \eta_1 z_1. \tag{12}$$

Equations (12) have solution provided the inequality $\eta^2 \leq \mu^2$ is satisfied. This inequality can be rewritten in the form $q^2 \leq q_{cr}^2$, $|\mu|<1$ or $q^2 \geq q_{cr}^2$, $|\mu|>1$. The former condition together with inequalities $q^2 \geq \varepsilon_i \mu_i$ leads to either Eq. (10b) or Eq. (10c), while the latter condition indicates that the wave vector should be greater than maximum of the three values $q^2 > \max\{q_{cr}^2, \varepsilon_1\mu_1, \varepsilon_2\mu_2\}$.

Interface between two different LHMs

As was mentioned above, the SW can exist in the case of equal signs of permeabilities, $\mathrm{sgn}(\mu_1) = \mathrm{sgn}(\mu_2)$. Thus both media can be chosen to be LHM, i.e., $\varepsilon_i<0$, $\mu_i<0$. An analysis of this case revealed the following constraints for the SW wave vector and system parameters

$$\varepsilon_1\mu_1 < \varepsilon_2\mu_2 < q^2 < q_{cr}^2, \quad \mu<1, \tag{13a}$$

$$\varepsilon_2\mu_2 < \varepsilon_1\mu_1 < q_{cr}^2 < q^2, \quad \mu>1, \tag{13b}$$

$$q_{cr}^2 < \varepsilon_1\mu_1 < \varepsilon_2\mu_2 < q^2, \quad \mu>1. \tag{13c}$$

We note that the surface mode in the case of two contacting LHM has energy flow in both media directed opposite to phase velocity [see Eq. (7)]. In the general case of the interface between focusing and defocusing media the total energy flow associated with the SW is

$$\langle S \rangle = \frac{cq\eta_1}{4\pi\mu_1\chi_1}\left[\left(1+\frac{\eta}{\mu^2\alpha}\right) + \left(1+\frac{1}{\mu\alpha}\right)\tanh(\eta_1 z_1)\right]. \tag{14}$$

V. INTERFACE BETWEEN LINEAR AND SELF-DEFOCUSING MEDIA ($\alpha_1=0, \chi_2<0$)

In this section we consider the case when medium 1 is linear. Because of the condition $\chi_2<0$ the solution (2b) must be taken in the medium 2. The amplitude $A_1(z)$ in the linear medium has the form $A_1(z) = a_1 e^{-\eta_1 z}$ and BCs lead to the following relations:

$$\tanh \eta_2 z_2 = -\frac{\eta}{\mu}, \tag{15a}$$

$$a_1^2 = a_2^2\left(\frac{\mu^2}{\eta^2} - 1\right) = 2\eta_1^2(\mu^2 - \eta^2)/|\chi_2|, \qquad (15b)$$

and for $z_2 > 0$, $\text{sgn}(a_1) = -\text{sgn}(a_2)$, $\text{sgn}(\mu_1) = -\text{sgn}(\mu_2)$. As a consequence of Eqs. (15) the condition $|\mu| \geqslant \eta$ is necessary for the mode to exist. The mode profile is qualitatively similar to the profile shown in Fig. 1(a) with the only difference that the mode profile in medium 1 is exponential rather than hyperbolic sine or cosine.

The total energy flow associated with this mode is

$$\langle S\rangle = \frac{cq\eta_1}{8\pi\mu_1|\chi_2|}(|\mu| - \eta)\left(\eta + |\mu| - \frac{2}{|\mu|}\right). \qquad (16)$$

The formulas (15) and (16) describe SWs at both the linear RHM/nonlinear LHM interface and linear LHM/nonlinear RHM interface.

Let us consider, e.g., the linear LHM/nonlinear RHM interface in more detail. Then $\varepsilon_1 < 0$, $\mu_1 < 0$, $\mu_2 > 0$, while ε_2 can be of either sign. It follows from Eq. (16) that $\langle S\rangle$ is negative if the condition

$$|\mu| > \eta > \mu_c = \frac{2}{|\mu|} - |\mu| \qquad (17)$$

is satisfied. This is possible only for $|\mu| > 1$. Equation (17) can be rewritten in the form $q^2 > \alpha_1$, $|\mu| \geqslant \sqrt{2}$ and $q^2 > \max\{\alpha_1, \alpha_2\}$, $1 < |\mu| < \sqrt{2}$, where $\alpha_1 = \mu_2(|\varepsilon_1| - \varepsilon_2)/(|\mu| - 1)$ and $\alpha_2 = (\mu_2\varepsilon_2 - \mu_1\varepsilon_1\mu_c)/(1 - \mu_c)$.

It is worth noting that for $q^2 = \varepsilon_i\mu_i$ Eq. (1) has the following algebraic singular solution:

$$A = \frac{a_i}{(z - z_i)}, \quad a_i = \pm\sqrt{2/|\chi_i|}. \qquad (18)$$

This solution is a limiting case of solutions (2b) when q^2 approaches $\varepsilon_i\mu_i > 0$ from above. Thus in this case the exponential decay of the mode field in the corresponding medium is turned into the algebraic one. This transformation can be realized for all considered modes. For instance, in the case of a linear LHM/nonlinear RHM interface the mode parameters are $z_2 = 1/|\mu|\eta_1$, $a_1 = \mu\eta_1 a_2$, $a_2 = \pm\sqrt{2/|\chi_2|}$, $q^2 = \varepsilon_2\mu_2 > \varepsilon_1\mu_1 > 0$. The energy flow $\langle S\rangle$ can be easily calculated by substituting $\eta = 0$ in Eq. (16).

VI. CONCLUSION

We have studied the properties of nonlinear TE-polarized surface modes at the interface between different conventional and left-handed media and between two LHM. The interfaces between two nonlinear media as well as between linear and nonlinear media have been considered. In the case of a nonlinear/nonlinear interface, two of the three possible combinations of nonlinear materials, namely defocusing/defocusing and focusing/defocusing interfaces, have been studied. Upon constructing the surface modes, the singular solutions of the corresponding governing equation have been utilized. The considered cases are complementary to the case of a focusing/focusing interface studied in [24], and together they cover all possible combinations of nonlinear Kerr-type materials. The Poynting vector of the surface modes has been calculated, and system parameter domains where the energy flow associated with SW is directed opposite to the phase velocity have been identified.

ACKNOWLEDGMENTS

S.D. gratefully acknowledges support from CNRS during his stay at the Institut Fresnel, Faculté des Sciences et Techniques de Saint Jérôme, Marseille and the hospitality of the Nanotech Institute, the University of Texas at Dallas.

[1] V. G. Veselago, Sov. Phys. Usp. **10**, 509 (1968).
[2] D. R. Smith and N. Kroll, Phys. Rev. Lett. **85**, 2933 (2000).
[3] R. A. Shelby, D. R. Smith, and S. Schultz, Science **292**, 77 (2001).
[4] C. G. Parazzoli, R. B. Greegor, K. Li, B. E. C. Koltenbah, and M. Tanielian, Phys. Rev. Lett. **90**, 107401 (2003).
[5] A. A. Houck, J. B. Brock, and I. L. Chuang, Phys. Rev. Lett. **90**, 137401 (2003).
[6] A. N. Lagarkov and V. N. Kissel, Phys. Rev. Lett. **92**, 077401 (2004); A. Grbic and G. V. Eleftheriades, *ibid.* **92**, 117403 (2004).
[7] J. B. Pendry and D. R. Smith, Phys. Today **57**, 37 (2004); also further reading can be found in a special edition of Opt. Express **11**, 639 (2003)and K. Yu. Bliokh and Yu. P. Bliokh, Phys. Usp. **174**, 439 (2004).
[8] L. I. Mandelstam, Zh. Eksp. Teor. Fiz. **15**, 475 (1945); also earlier in The 4th lecture given at Moscow State University (05/05/1944). Collection of Science papers, Vol. 2 (Nauka, Moscow).
[9] V. M. Agranovich and V. L. Ginzburg, in *Crystal Optics with Spatial Dispersion and Excitons* (Springer, Berlin, 1984).
[10] M. Notomi, Phys. Rev. B **62**, 10696 (2000).
[11] P. Parimi, W. T. Lu, P. Vodo, and S. Sridhar, Nature (London) **426**, 404 (2003).
[12] V. M. Agranovich, R. H. Baughman, Y. R. Shen, and A. A. Zakhidov, J. Lumin. **110**, 167 (2004).
[13] V. A. Podolskiy, A. K. Sarychev, and V. M. Shalaev, Opt. Express **11**, 735 (2003).
[14] V. M. Agranovich, Y. R. Shen, R. H. Baughman, and A. A. Zakhidov, Phys. Rev. B **69**, 165112 (2004).
[15] M. W. McCall, A. Lakhtakia, and W. S. Weighofer, Eur. J. Phys. **23**, 353 (2002).
[16] A. A. Zharov, I. V. Shadrivov, and Yu. S. Kivshar, Phys. Rev. Lett. **91**, 037401 (2003).
[17] H. Raether, *Surface Plasmons* (Springer-Verlag, Heidelberg, 1988).
[18] R. E. Camley and D. L. Mills, Phys. Rev. B **26**, 1280 (1982).
[19] R. Ruppin, Phys. Lett. A **277**, 61 (2000).

[20] Yu. I. Bespyatykh, A. S. Bugaev and I. E. Dikshtein, Phys. Solid State **43**, 2130 (2001).
[21] S. A. Darmanyan, M. Nevière, and A. A. Zakhidov, Opt. Commun. **225** 233 (2003).
[22] L. Chen, S. He, and L. Shen, Phys. Rev. Lett. **92**, 107404 (2004).
[23] M. W. Feise and Yu. S. Kivshar, Phys. Lett. A **334**, 326 (2005).
[24] I. V. Shadrivov, A. A. Sukhorukov, Yu. S. Kivshar, A. A. Zharov, A. D. Boardman, and P. Egan, Phys. Rev. E **69**, 016617 (2004).
[25] G. I. Stegeman, C. T. Seaton, J. Ariyasu, R. F. Wallis, and A. A. Maradudin, J. Appl. Phys. **58**, 2453 (1985).
[26] S. Darmanyan and M. Neviere, Phys. Lett. A **281**, 260 (2001).